

# Power Analysis

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# Power Analysis

## What is Power In Testing The Difference Between 2 Proportions

Most of my experience has been in automotive direct marketing.

Example

Direct mail for automotive

- Sent mail with incentive
- Sent email

Send mail and ask did the mail help sell cars? Did it sell enough cars to make a profit?

How to ascertain if the mail worked? Treated group and a control group.

Test difference in buy rates and estimate **incremental sales** – ( treated purchase rate – control purchase rate) x N of treated

For testing we need a significance level

In marketing 90% confidence is common - It means that there is only 1 chance in 10 that the differences we see occur by chance

Marketing people want to mail everyone, sometimes you need to convince them to have a control group

Usually the N of the mailing is determined by budget.

So the usual question is how many people are going to be in the control group?

# Power Analysis

## What is Power In Testing The Difference Between 2 Proportions

Power is the probability that, given the significance level we will accept,  $p_1$ ,  $p_2$ ,  $n_1$ , and  $n_2$  we will find a difference large enough that we will call it statistically significant

Why is power important?

- If you're performing a test and you have a low probability of finding a difference
  1. Why go through the effort and expense
  2. Low power could cause you to miss an important difference
  3. Underpowered tests build a history of failure with your client

Hypothesis Testing

Alpha –  $p$  of calling a difference if the isn't one – false positive

Beta – the  $p$  of saying there is no difference when there is one – false negative

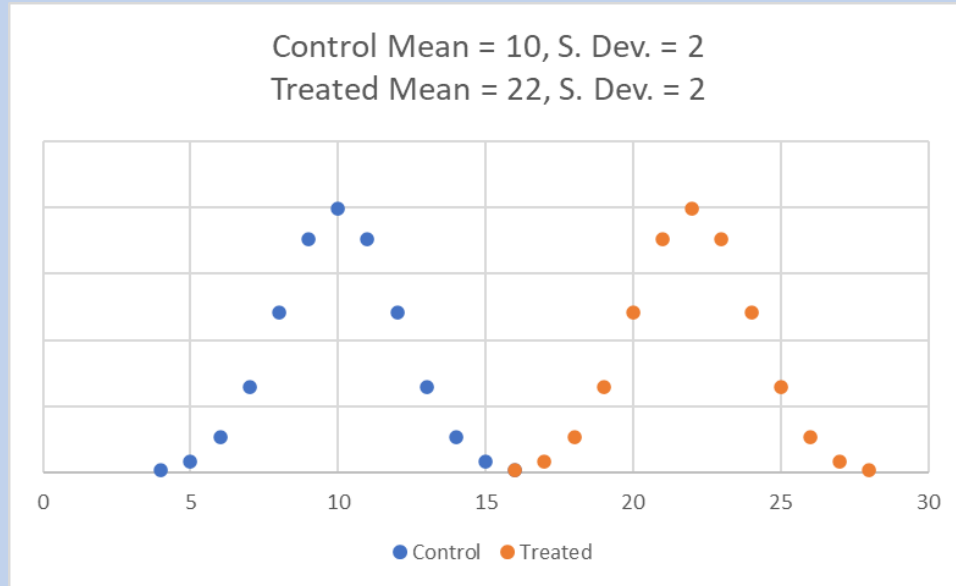
$1 - \beta$  - power

What influences power?

- $D$  – effect size - the difference between groups
- Variance - the variability within the samples
- $N$  – the number of observations in each group
- Alpha – the significance level you wish to achieve
- One tailed or two tailed test

# Power Analysis

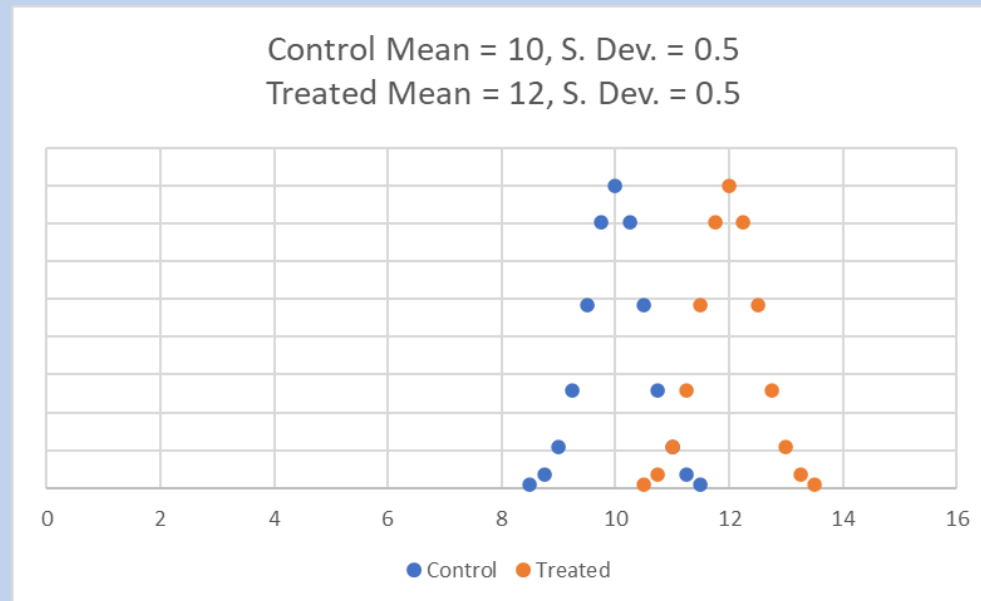
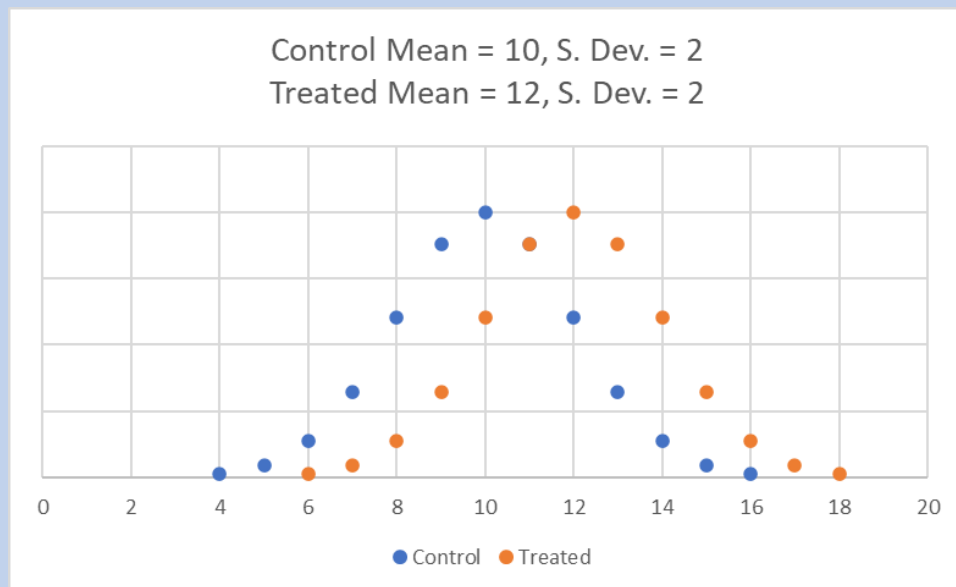
## What is Power In Testing The Difference Between 2 Proportions



Left – distributions of treated and control that do not overlap  
D is large

Lower left – D is made smaller and it is unlikely the difference will be called significant

Lower right – same D but the variance is reduced because I increased N



# Power Analysis

$p_1$  and  $p_2$  in this example are buy rates – the proportion of owners who purchased a vehicle

Where does  $p_1$  and  $p_2$  come from?

- Experience
- Projection – decide how much of a difference is worth finding

Where do we get the significance level?

What about  $n_1$  and  $n_2$ ?

Usually we choose a power level we want to achieve and calculate the  $n$  required to get there

In this case we have a fixed  $N$  and are looking to see how many we need to assign to the control group

# Power Analysis

```
data _null_;  
  controlp = .0063; mailp = controlp * 1.05;  
  call symput('control',controlp); call symput('treated',mailp);  
run;  
  
proc power ;  
  twosamplefreq /* specify the test you're interested in */  
  power = . /* set the parameter you want calculated to missing */  
  alpha = .1 /* set alpha */  
  sides = U /* specify 1 tail or two tail test */  
  groupproportions = (&control,&treated) /* p of control and treated */  
  ntotal=1075494 /* total N */  
  groupweights= (1 9) (2 8) (25 75) (3 7) (4 6) (5 5) ; /proportion in each group */  
;  
run;
```

**Note: this treated and control difference could result in the sale of an additional 338 vehicles**

# Power Analysis In Direct Marketing

## 5% improvement Treated vs. Control

Fixed Scenario Elements	
Distribution	Asymptotic normal
Method	Normal approximation
Number of Sides	U
Alpha	0.1
Group 1 Proportion	0.0063
Group 2 Proportion	0.006615
Nominal Total Sample Size	1075494
Null Proportion Difference	0

Computed Power				
Index	Weight1	Weight2	Actual N Total	Power
1	1	9	1075490	0.472
2	2	8	1075490	0.634
3	25	75	1075400	0.684
4	3	7	1075490	0.720
5	4	6	1075490	0.763
6	5	5	1075490	0.776

## 10% improvement Treated vs. Control

Fixed Scenario Elements	
Distribution	Asymptotic normal
Method	Normal approximation
Number of Sides	U
Alpha	0.1
Group 1 Proportion	0.0063
Group 2 Proportion	0.00693
Nominal Total Sample Size	1075494
Null Proportion Difference	0

Computed Power				
Index	Weight1	Weight2	Actual N Total	Power
1	1	9	1075490	0.872
2	2	8	1075490	0.975
3	25	75	1075400	0.987
4	3	7	1075490	0.992
5	4	6	1075490	0.996
6	5	5	1075490	0.997

# Power Analysis In Direct Marketing

```
proc power ;  
twosamplefreq  
power = .      /* calculate power */  
alpha = .1  
sides = U  
groupproportions = (0.0063, 0.006615)      /*5% improvement in sales */  
ntotal= 500000 1000000 1500000 2000000      /*vary the total sample size */  
groupweights= (1 9) (2 8) (25 75) (3 7) (5 5) ;      /* same group weights as before */  
plot x = n YOPTS=(REF=0.8 0.9 ) vary(color by groupweights);  
      /* request a plot x= parameter on x axis, yopts is asking for a reference line, request  
different colors for groupweights lines*/  
run;
```



# Power Analysis In Direct Marketing

