# Random Effects vs. Marginal Models: Different Approaches to Analyzing Repeated Measures / Longitudinal Data

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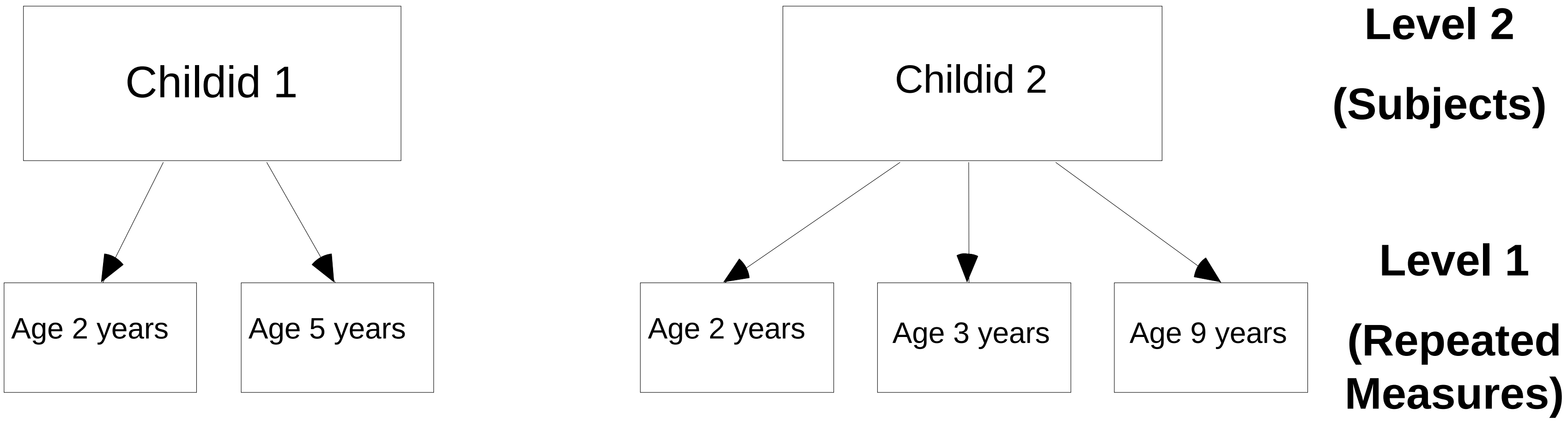
# Background:
# What is a Linear Mixed Model (LMM)?

- A parametric linear model for
  - Clustered data
  - Repeated Measures / Longitudinal data
- Continuous response
- Predictors may be
  - Fixed
  - Random
- This presentation will focus on an analysis of a longitudinal data set.

# Repeated Measures / Longitudinal Data

- **Longitudinal Data:**
  - Dependent variable measured multiple times for each unit of analysis, basically a type of repeated measures data
  - Repeated measures factor is time
  - Time may be over an extended period (e.g. years)
- Example
  - Autistic children measured at different ages
- Dropout may be a problem
- Missing data at some time points may be a problem (not a problem if MAR)

# Example of Repeated Measures/ Longitudinal Data Structure



Childid 1

Childid 2

Level 2

(Subjects)

Age 2 years

Age 5 years

Age 2 years

Age 3 years

Age 9 years

Level 1

(Repeated Measures)

•Each subject measured more than once

•Number of measurements does not need to be equal for all subjects

•Spacing of intervals does not have to be equal for all measurement times

# Fixed Factor

- Fixed Factor: A categorical/classification variable
  - all levels of interest are included
    - Treatment level
    - Gender
- Levels of fixed factors can be defined to represent contrasts of interest
  - Female vs. Male
  - High Dose vs. Control, Medium Dose vs. Control

# Random Factor

- Random Factor: A classification variable
  - Levels can be thought of as being randomly sampled from a population
    - Classroom
    - Subject
- Variation in the dependent variable across levels of the random factor can be estimated and assessed
- Usually, random factors do not represent conditions chosen to meet the needs of the study
- Results can be generalized to a greater population

# Fixed **Effects**

- Also called regression coefficients or fixed-effect parameters
  - Describe the relationship between the dependent variable and predictor variables for an entire population
- Represented as unknown **fixed** quantities ($\beta$) in a LMM
  - The value of a given $\beta$ does not vary across subjects
- $\beta$ is estimated based on data

# Random **Effects**

- Random values associated with levels of a random factor
- Represented as random variables ($\boldsymbol{u_i}$ for the *i*th subject*)* in a LMM
  - Specific to a given level of a random factor
  - Vary across subjects
    - Classroom-specific intercepts in a clustered design
    - Subject-specific intercepts in a repeated measures design
- Usually describe random deviations in the relationships described by fixed effects
- Can be for categorical or continuous variables
  - Random intercepts
  - Random slopes

# General Specification of an LMM for the *i*th Subject:

$$\boldsymbol{Y}_i = \underbrace{\boldsymbol{X}_i \boldsymbol{\beta}}_{\text{fixed}} + \underbrace{\boldsymbol{Z}_i \boldsymbol{u}_i + \boldsymbol{\varepsilon}_i}_{\text{random}}$$

$$\boldsymbol{u}_i \sim N(\boldsymbol{0}, \boldsymbol{D})$$

$$\boldsymbol{\varepsilon}_i \sim N(\boldsymbol{0}, \boldsymbol{R}_i)$$

Called a **mixed model** because it has a mix of fixed (***β)*** and random (***u$_i$)*** effects.
Both ***D*** and ***R$_i$*** are variance-covariance matrices, and as such, are required to be positive-definite

# The *D* Matrix

Variance-covariance matrix for the *q* random effects ($\boldsymbol{u}_i$) for the ith subject. SAS calls this the *G* matrix and defines it for all subjects, rather than for individuals.

$$\boldsymbol{D} = Var(\boldsymbol{u}_i) = \begin{pmatrix} Var(u_{1i}) & cov(u_{1i}, u_{2i}) & \cdots & cov(u_{1i}, u_{qi}) \\ cov(u_{1i}, u_{2i}) & Var(u_{2i}) & \cdots & cov(u_{2i}, u_{qi}) \\ \vdots & \vdots & \ddots & \vdots \\ cov(u_{1i}, u_{qi}) & cov(u_{2i}, u_{qi}) & \cdots & Var(u_{qi}) \end{pmatrix}$$

For Example: If there were only one random effect per subject (e.g., a random intercept), then ***D*** would be a 1 X 1 matrix.

If there were two random effects per subject, e.g., a random intercept and a random slope, then ***D*** would be 2 X 2.

# Two Common Structures for $\boldsymbol{D}$

Many different structures for ***D*** are possible:

**Variance components**

**type=vc**

$$\boldsymbol{D} = Var(\boldsymbol{u}_i) = \begin{pmatrix} \sigma_{u1}^2 & 0 \\ 0 & \sigma_{u2}^2 \end{pmatrix}$$

**Unstructured**

**type=un**

$$\boldsymbol{D} = \begin{pmatrix} \sigma_{u1}^2 & \sigma_{u1,u2} \\ \sigma_{u1,u2} & \sigma_{u2}^2 \end{pmatrix}$$

Note: In these examples, we have two random effects defined for each subject. The diagonal elements represent variances of the random effects; the off-diagonal elements represent covariances between the random effects

# The $\boldsymbol{R}$ Matrix

Variance-covariance matrix for the $n_i$ **residuals** (**$\boldsymbol{\varepsilon}_i$**) for the ith subject

$$\boldsymbol{R}_i = Var(\boldsymbol{\varepsilon}_i) = \begin{pmatrix} Var(\varepsilon_{1i}) & cov(\varepsilon_{1i}, \varepsilon_{2i}) & \cdots & cov(\varepsilon_{1i}, \varepsilon_{n_i i}) \\ cov(\varepsilon_{1i}, \varepsilon_{2i}) & Var(\varepsilon_{2i}) & \cdots & cov(\varepsilon_{2i}, \varepsilon_{n_i i}) \\ \vdots & \vdots & \ddots & \vdots \\ cov(\varepsilon_{1i}, \varepsilon_{n_i i}) & cov(\varepsilon_{2i}, \varepsilon_{n_i i}) & \cdots & Var(\varepsilon_{n_i i}) \end{pmatrix}$$

Note: The dimension of $\boldsymbol{R_i}$ depends on the number of observations ($n_i$) for subject *i*. For a subject with 5 repeated measures, the $\boldsymbol{R_i}$ matrix would be 5 X 5.

# Some Commonly Used Structures for ***R***

**Unstructured**
**type = UN**

$$\begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{pmatrix}$$

**Variance**
**Components**
**type=VC**

$$\begin{pmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{pmatrix}$$

**Compound Symmetry**
**type = CS**

$$\begin{pmatrix} \sigma^2 + \sigma_1 & \sigma_1 & \sigma_1 \\ \sigma_1 & \sigma^2 + \sigma_1 & \sigma_1 \\ \sigma_1 & \sigma_1 & \sigma^2 + \sigma_1 \end{pmatrix}$$

**Banded**
**type = UN(2)**

$$\begin{pmatrix} \sigma_1^2 & \sigma_{12} & 0 \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} \\ 0 & \sigma_{23} & \sigma_3^2 \end{pmatrix}$$

# More structures for *R*

**First-order Autoregressive**

**type = AR(1)**

$$\begin{pmatrix} \sigma^2 & \rho\sigma^2 & \rho^2\sigma^2 \\ \rho\sigma^2 & \sigma^2 & \rho\sigma^2 \\ \rho^2\sigma^2 & \rho\sigma^2 & \sigma^2 \end{pmatrix}$$

**Toeplitz**

**type = Toep**

$$\begin{pmatrix} \sigma^2 & \sigma_1 & \sigma_2 \\ \sigma_1 & \sigma^2 & \sigma_1 \\ \sigma_2 & \sigma_1 & \sigma^2 \end{pmatrix}$$

**Toeplitz (2)**

**type = Toep(2)**

$$\begin{pmatrix} \sigma^2 & \sigma_1 & 0 \\ \sigma_1 & \sigma^2 & \sigma_1 \\ 0 & \sigma_1 & \sigma^2 \end{pmatrix}$$

**Heterogeneous**

**Compound Symmetry**

**type = CSH**

$$\begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \rho\sigma_1\sigma_3 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 & \rho\sigma_2\sigma_3 \\ \rho\sigma_1\sigma_3 & \rho\sigma_2\sigma_3 & \sigma_3^2 \end{pmatrix}$$

**Heterogeneous 1st-order**

**Autoregressive**

**type = ARH(1)**

$$\begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \rho^2\sigma_1\sigma_3 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 & \rho\sigma_2\sigma_3 \\ \rho^2\sigma_1\sigma_3 & \rho\sigma_2\sigma_3 & \sigma_3^2 \end{pmatrix}$$

**Heterogeneous Toeplitz**

**type = Toeph**

$$\begin{pmatrix} \sigma_1^2 & \rho_1\sigma_1\sigma_2 & \rho_2\sigma_1\sigma_3 \\ \rho_1\sigma_1\sigma_2 & \sigma_2^2 & \rho_1\sigma_2\sigma_3 \\ \rho_2\sigma_1\sigma_3 & \rho_1\sigma_2\sigma_3 & \sigma_3^2 \end{pmatrix}$$

# Covariance Parameters

- We estimate a set of covariance **parameters** for the variance-covariance matrices, ***D*** and ***R***.
  - For ***D*** we estimate $\theta_{\boldsymbol{D}}$
  - For ***R*** we estimate $\theta_{\boldsymbol{R}}$
- The number of covariance parameters that we estimate depends on the structure we specify for ***D*** and ***R***.

# Marginal Model vs. LMM

- **LMM** uses random effects explicitly to explain between-subject variance
  - Subject-specific model
- **Marginal model** does not use random effects in its specification at all
  - Population-averaged model
- **Implied marginal model**
  - Marginal model that results from fitting a LMM

# A Strictly Marginal Model
# With no random effects

$$\boldsymbol{Y}_i = \boldsymbol{X}_i \boldsymbol{\beta} + \boldsymbol{\varepsilon}_i^*$$

$$\boldsymbol{\varepsilon}_i^* \sim N(\mathbf{0}, \boldsymbol{V}_i)$$

$$\boldsymbol{V}_i = \boldsymbol{R}_i$$

***V***$_{\text{i}}$ is the marginal variance-covariance matrix for ***Y***$_{\text{i}}$
In this marginal model, we do not specify any random effects.
There is no ***G*** matrix in this model.
Covariances, and hence correlations, among residuals
are specified directly through the ***R***$_{\text{i}}$ matrix

# Implied Marginal Distribution of ***Y***i Based on a LMM

$$\boldsymbol{Y}_i \sim N(\boldsymbol{X}_i\boldsymbol{\beta},\ \ \boldsymbol{Z}_i\boldsymbol{D}\boldsymbol{Z}_i' + \boldsymbol{R}_i)$$

$$E(\boldsymbol{Y}_i) = \boldsymbol{X}_i\boldsymbol{\beta}$$

$$Var(\boldsymbol{Y}_i) = \boldsymbol{V}_i = \boldsymbol{Z}_i\boldsymbol{D}\boldsymbol{Z}_i' + \boldsymbol{R}_i\,\boldsymbol{.}$$

In the **implied marginal model**, ***V****ᵢ* is formed from ***D*** and ***R****ᵢ*, but while***V****ᵢ* is required to be positive-definite, ***D*** and ***R****ᵢ* are not.

# Model Fit: Akaike Information Criteria (AIC)

- SAS calculates the AIC based on the (ML or REML) log likelihood, as shown below:

$$AIC = -2 \times l(\hat{\beta}, \hat{\theta}) + 2p$$

- The penalty is 2p, where p represents the total number of parameters being estimated for both the fixed and random effects.
- Can be used to compare two models fit for the same observations, models need not be nested.
- Smaller is better.

# Model Fit: Bayes Information Criterion (BIC)

- BIC applies a greater penalty for models with more parameters than does AIC.

$$BIC = -2 \times l(\hat{\beta}, \hat{\theta}) + p \times \ln(n)$$

- The penalty to the likelihood is number of parameters, p, times ln(n), where n is the total number of observations in the data set.
- Can be used to compare two models for the same observations, need not be nested.
- Smaller is better.

# Repeated Measures / Longitudinal Data Setup

- Data are in Long Form, one row for each repeated measurement on each subject
- Each row contains:
  - Information on the repeated measurements
    - Dependent variable
    - Time-varying covariates to be included in the model
  - Plus information on the subject / unit of analysis
    - Unit / subject ID
    - Time-invariant covariates to be included in the model
    - These are repeated for each row of data for a subject

# Proc Mixed Syntax

- **Model** statement specifies the fixed factors and covariates in the model
- **Random** statement specifies the random effects to be included in the model, and specifies the structure of the ***D*** matrix of variances and covariances for the random effects (called ***G*** matrix by SAS)
- **Repeated** statement specifies the structure of the residual covariance matrix, ***R***

# The Autism Data Set

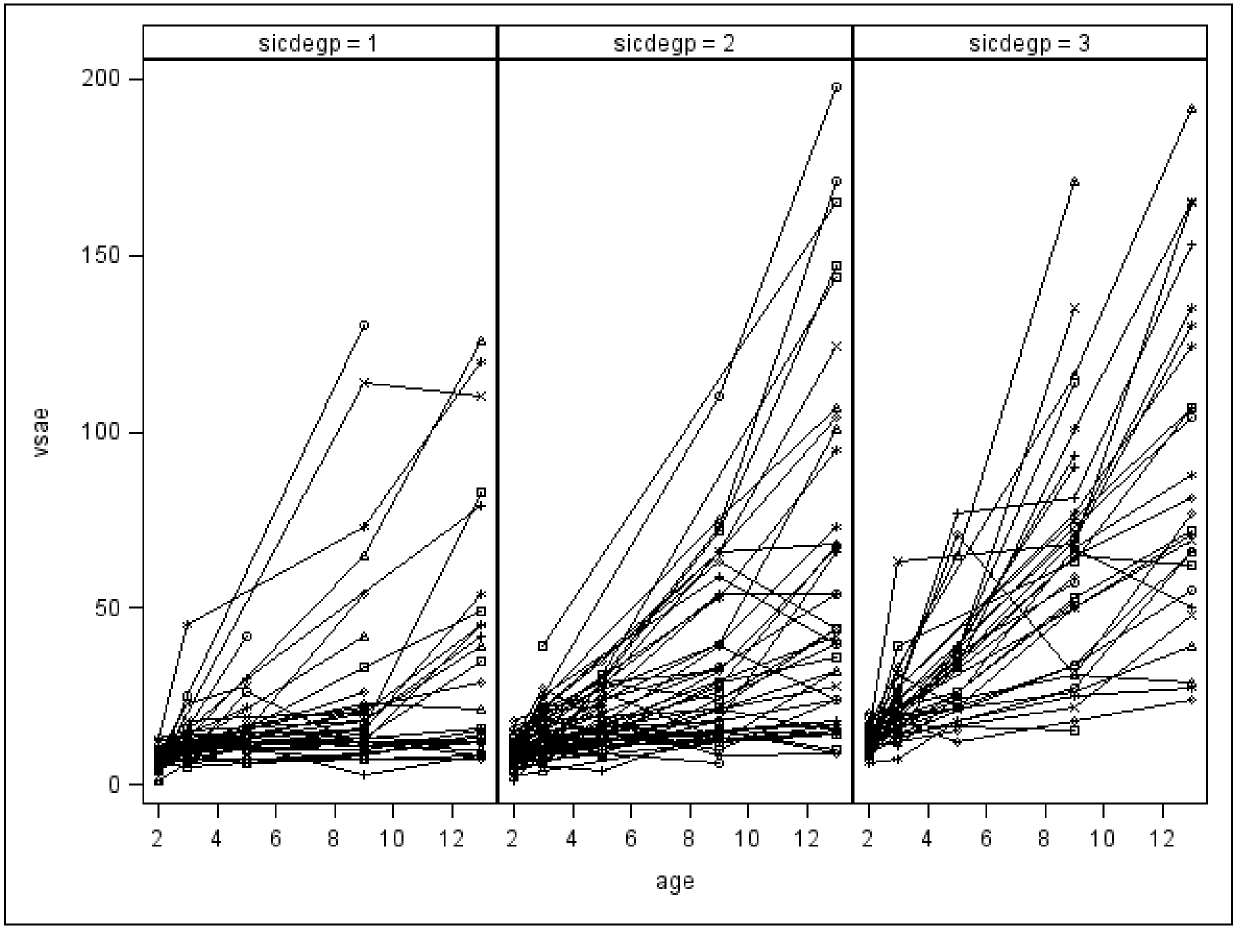
- **autism.csv** This data set was derived from a study of 158 children with Autism Spectrum Disorder (Oti, Anderson, Lord, 2006).
- Measurements were made at five basic ages for each child: 2, 3, 5, 9, and 13 years. Not all children were measured at all time points.
- We will analyze VSAE, a measure of socialization, for these children as a function of their expressive skills (SICDEGP) measured at baseline (time invariant), and their current age (time-varying).

# Structure of Autism.csv data set

```
age,vsae,sicdegp,childid
2,6,3,1
3,7,3,1
5,18,3,1
9,25,3,1
13,27,3,1
2,17,3,3
3,18,3,3
5,12,3,3
9,18,3,3
13,24,3,3
2,12,3,4
3,14,3,4
5,38,3,4
9,114,3,4
```

```
Obs        childid        age      sicdegp

  1           1             2          3
  2           1             3          3
  3           1             5          3
  4           1             9          3
  5           1            13          3
  6           3             2          3
  7           3             3          3
  8           3             5          3
  9           3             9          3
 10           3            13          3
 11           4             2          3
 12           4             3          3
 13           4             5          3
 14           4             9          3
```

# Plots of VSAE Over Time for Each Child by Baseline Expressive Language Group



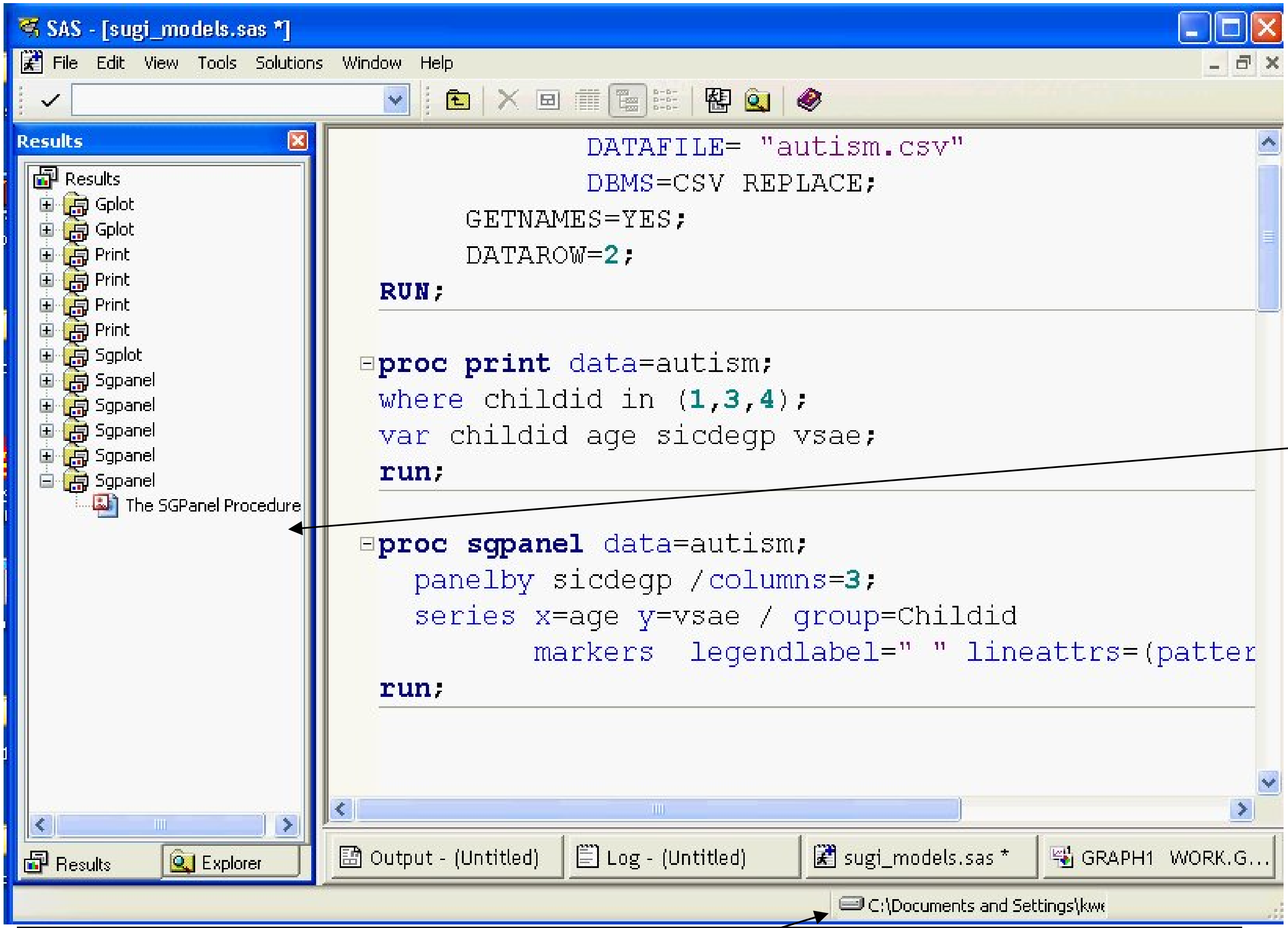
# Proc Sgpanel code for Individual line Graphs (SAS 9.2)

```
proc sgpanel data=autism;
  panelby sicdegp /columns=3;
  series x=age y=vsae / group=Childid
         markers  legendlabel=" " lineattrs=(pattern=1 color=black);
run;
```

The statistical graphics in SAS 9.2 are terrific. I’m still experimenting.

To get help, go to “SAS Help and Documentation”….SAS Products…SAS/Graph…Statistical Graphics Procedures.

# Location of .png file from Sgpanel

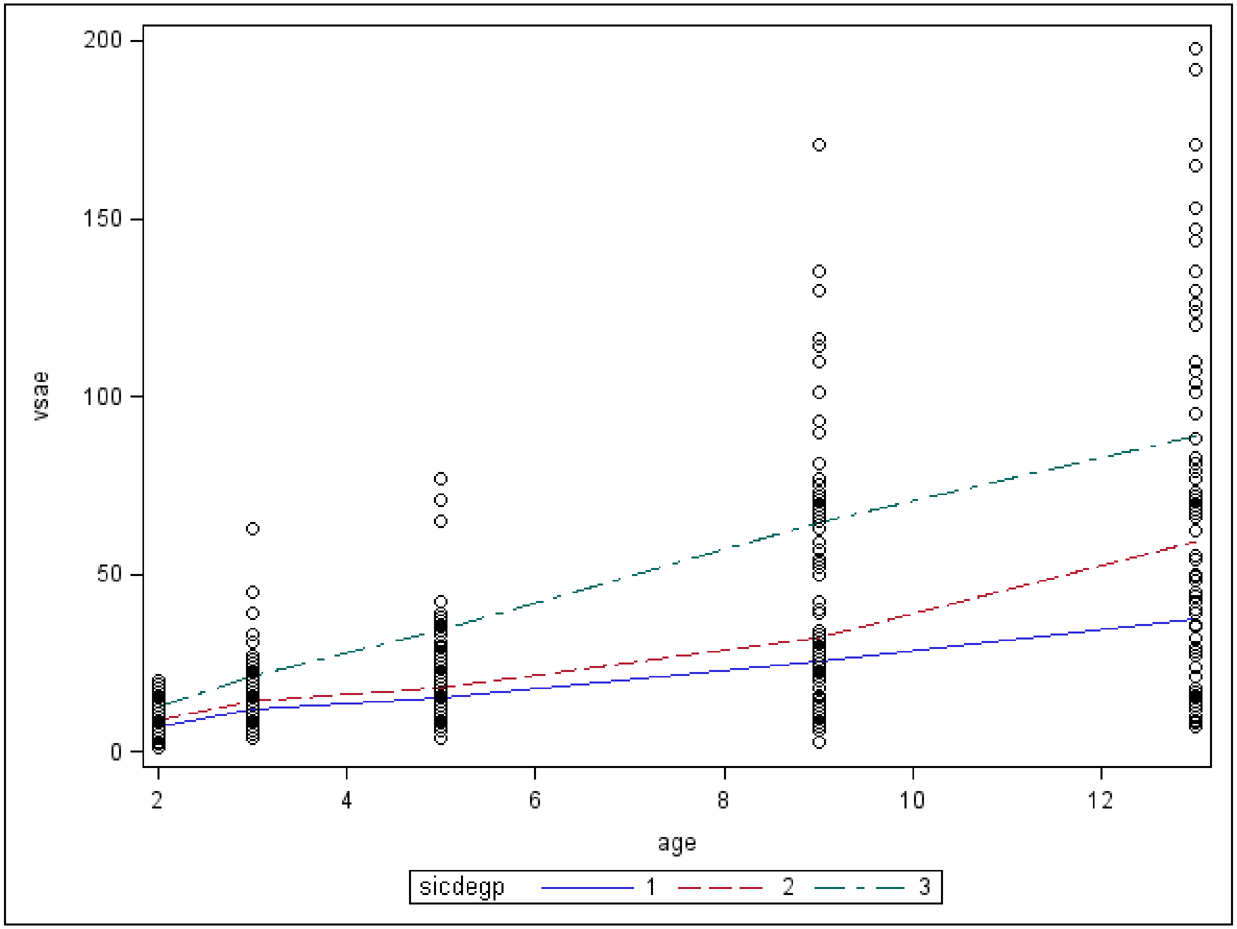


```
            DATAFILE= "autism.csv"
            DBMS=CSV REPLACE;
     GETNAMES=YES;
     DATAROW=2;
RUN;

proc print data=autism;
where childid in (1,3,4);
var childid age sicdegp vsae;
run;

proc sgpanel data=autism;
  panelby sicdegp /columns=3;
  series x=age y=vsae / group=Childid
         markers  legendlabel=" " lineattrs=(patter
run;
```

Graph is in results window, not Graph window.

Graph will be saved in default folder as a .png file

# Mean Profiles by SICD Group



# SAS Sgplot Code for Mean Plots by SICD Group

```
proc sort data=autism;
by sicdegp age;
run;
proc means data=autism noprint;
  by sicdegp age;
  output out=meandat mean(VSAE)=mean_VSAE;
run;
data autism2;
  merge autism meandat(drop=_type_ _freq_);
  by sicdegp age;
run;
proc sgplot data=autism2;
  series x=age y=mean_VSAE / group=SICDEGP;
  scatter x=age y=VSAE ;
run;
```

# Discussion of Plots

- There is substantial variation in VSAE scores between children, and this variation gets larger over time.
- Although some children’s scores do not seem to increase, there is a generally increasing trend in the means of VSAE over time in all three SICD groups.
- It looks like there may be a quadratic trend in mean VSAE scores, especially in group two.

# Modeling Strategy

- We first attempt to fit a LMM with 3 random effects for each subject: a random intercept, random slope for AGE, and random quadratic effect of AGE.
  - This is known as a random coefficients, growth-curve, or Laird-Ware Model
- We then fit an implied marginal model, in which we relax constraints on the variance-covariance matrices, $\boldsymbol{D}$ and $\boldsymbol{R}_i$
- Finally, we fit a strictly marginal model.

# LMM with Random Child-Specific Intercepts, Slopes and Quadratic Effects

**LMM:**

$$\left.\begin{array}{l}
\text{VSAE}_{ti} = \beta_0 + \beta_1 \times \text{AGE\_2}_{ti} + \beta_2 \times \text{AGE\_2SQ}_{ti} + \beta_3 \times \text{SICDEGP1}_i \\
+\beta_4 \times \text{SICDEGP2}_i + \beta_5 \times \text{AGE\_2}_{ti} \times \text{SICDEGP1}_i \\
+\beta_6 \times \text{AGE\_2}_{ti} \times \text{SICDEGP2}_i + \beta_7 \times \text{AGE\_2SQ}_{ti} \times \text{SICDEGP1}_i \\
+\beta_8 \times \text{AGE\_2SQ}_{ti} \times \text{SICDEGP2}_i +
\end{array}\right\} \textbf{fixed}$$

$$\left. u_{0i} + u_{1i} \times \text{AGE\_2}_{ti} + u_{2i} \times \text{AGE\_2SQ}_{ti} + \varepsilon_{ti} \right\} \textbf{random}$$

We include the **fixed effects** of AGE, AGE-Squared, SICDEGP, and interactions between AGE, AGE-Squared and SICDEGP.

We also include three **random effects** for each child: the intercept ($u_{0i}$), the linear slope of AGE ($\mathrm{u}_{1i}$), and the quadratic effect of AGE ($\boldsymbol{u}_{2i}$), to capture between-child variability.

# SAS Code for LMM

```
proc mixed data=autism2;
   class childid sicdegp;
   model vsae = age_2 age_2sq sicdegp age_2*sicdegp
          age_2sq*sicdegp / solution ddfm=sat;
   random int age_2 age_2sq
                             / subject=childid type=un g v ;
run;
```

# Distribution of Random Effects for the LMM

$$\boldsymbol{u}_i = \begin{pmatrix} u_{0i} \\ u_{1i} \\ u_{2i} \end{pmatrix} \sim N(\mathbf{0}, \boldsymbol{D}).$$

$$\boldsymbol{D} = \begin{pmatrix} \sigma_{int}^2 & \sigma_{int,age} & \sigma_{int,age\text{-}squared} \\ \sigma_{int,age} & \sigma_{age}^2 & \sigma_{age,age\text{-}squared} \\ \sigma_{int,age\text{-}squared} & \sigma_{age,age\text{-}squared} & \sigma_{age\text{-}squared}^2 \end{pmatrix}, \qquad \varepsilon_{ti} \sim N(0, \sigma^2)$$

We specify an unstructured ***D*** matrix for the random effects. There are 6 covariance parameters in the ***D*** matrix for the 3 random effects.

Note: There is no ***R*** matrix specified for this model, so ***R*** is assumed to be $\boldsymbol{\sigma^2 I}$

# LMM: Problem with ***G*** Matrix

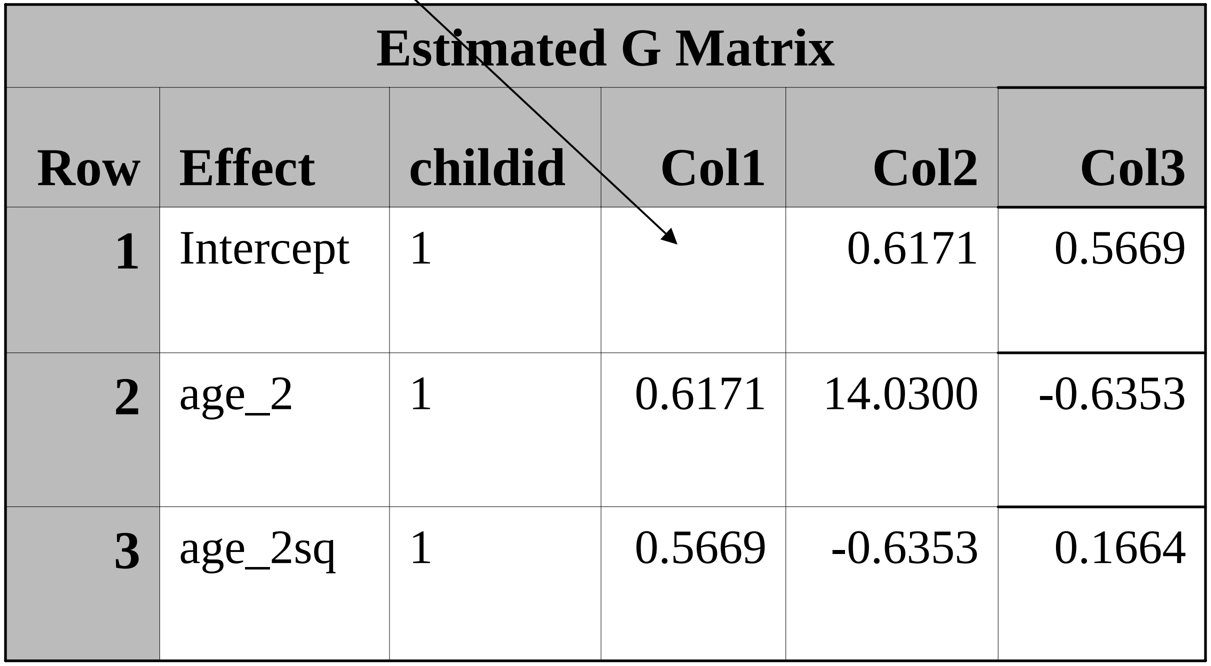
SAS reports problems fitting Model 6.1. We see the following note in the SAS log:

```
NOTE: Convergence criteria met.

NOTE: Estimated G matrix is not positive definite.

NOTE: Asymptotic variance matrix of covariance parameter estimates has
been found to be singular and a generalized inverse was used. Covariance
parameters with zero variance do not contribute to degrees of freedom
computed by DDFM=SATTERTH.
```

**We have a problem with the *G* matrix (referred to as the *D* matrix in this presentation). We need to investigate this problem.**

# LMM: SAS Output for ***G*** Matrix

We see that the value in the G matrix corresponding to the variance of the random intercepts is blank here.



**Estimated G Matrix**

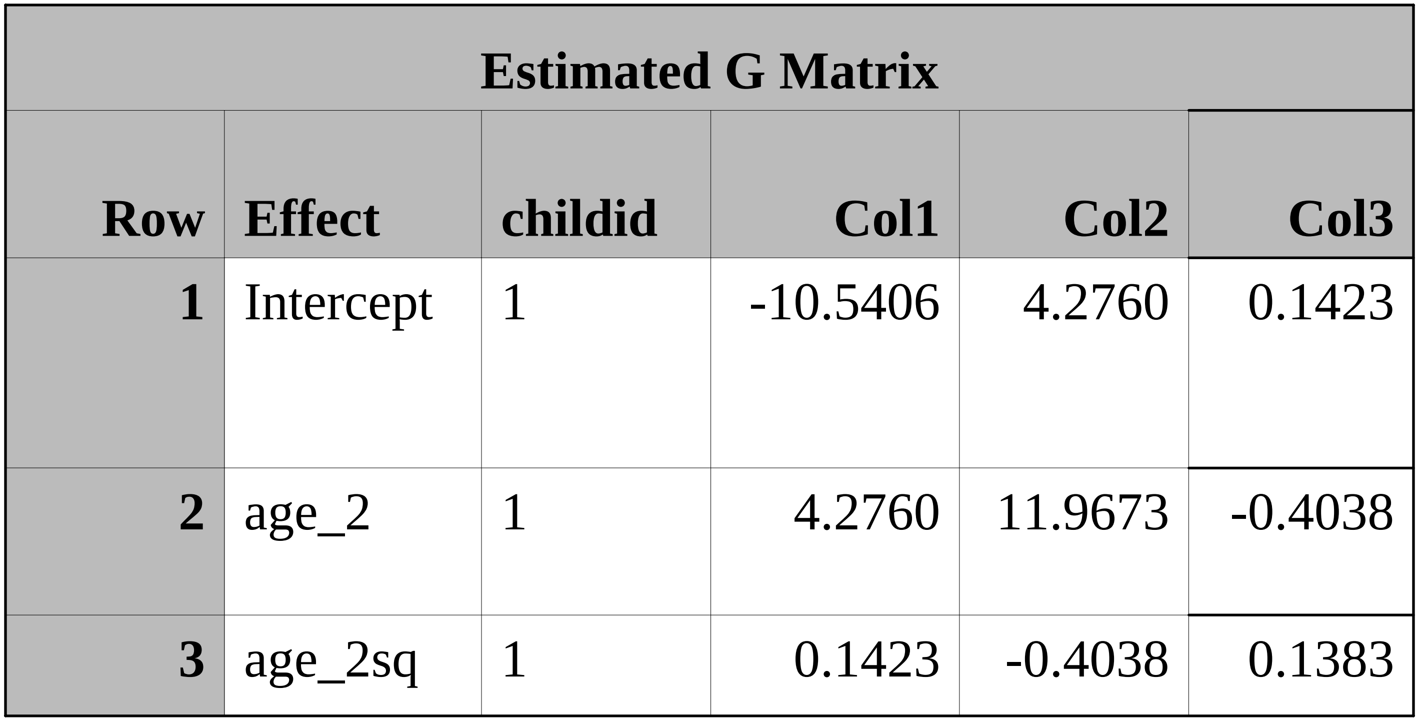
| Row | Effect | childid | Col1 | Col2 | Col3 |
|---|---|---|---|---|---|
| **1** | Intercept | 1 |  | 0.6171 | 0.5669 |
| **2** | age_2 | 1 | 0.6171 | 14.0300 | -0.6353 |
| **3** | age_2sq | 1 | 0.5669 | -0.6353 | 0.1664 |

# Fit the Implied Marginal Model

- We now refit the model
- Use the **nobound** option, to get the implied marginal model,
- Positive definiteness constraints on ***G*** and ***R*** are relaxed.

```
proc mixed data=autism2  nobound;
```

# Look at Unconstrained ***G*** Matrix



| Estimated G Matrix | | | | | |
|---|---|---|---|---|---|
| **Row** | **Effect** | **childid** | **Col1** | **Col2** | **Col3** |
| **1** | Intercept | 1 | -10.5406 | 4.2760 | 0.1423 |
| **2** | age_2 | 1 | 4.2760 | 11.9673 | -0.4038 |
| **3** | age_2sq | 1 | 0.1423 | -0.4038 | 0.1383 |

The new estimate of the variance of the random intercepts is **negative**!

Clearly, this LMM is not working, as a negative variance is impossible!

# Revised LMM: Remove the Random Intercept

```
proc mixed data = autism2;
   class childid sicdegp;
   model vsae = sicdegp age_2 age_2sq age_2*sicdegp
                age_2sq*sicdegp /
                solution ddfm=sat influence;
   random age_2 age_2sq /
                          subject = childid solution g v vcorr type = un;
run;
```

**Note: the only change in the model is that “int” has been deleted from the random statement.**

# G Matrix for Revised LMM without Random Intercept

There are no error messages in the log.

The 2x2 G matrix is positive-definite.

**Estimated G Matrix**

| Row | Effect | childid | Col1 | Col2 |
|---|---|---|---|---|
| 1 | age_2 | 1 | 14.6674 | -0.4401 |
| 2 | age_2sq | 1 | -0.4401 | 0.1315 |

**Covariance Parameter Estimates**

| Cov Parm | Subject | Estimate |
|---|---|---|
| UN(1,1) | childid | 14.6674 |
| UN(2,1) | childid | -0.4401 |
| UN(2,2) | childid | 0.1315 |
| Residual | | 38.4988 |

# *V* Matrix for the Revised LMM

The only variability in the intercept is the estimated residual variance.

There is no covariance between the Y values at baseline with any other ages

**Estimated V Matrix for childid 1**

| **Row** | **Col1** | **Col2** | **Col3** | **Col4** | **Col5** |
|---:|---:|---:|---:|---:|---:|
| **1** | 38.4988 | | | | |
| **2** | | **52.4175** | 39.9043 | 84.4687 | 119.16 |
| **3** | | 39.9043 | **157.39** | 273.57 | 423.87 |
| **4** | | 84.4687 | 273.57 | **770.95** | 1298.89 |
| **5** | | 119.16 | 423.87 | 1298.89 | **2566.53** |

# Alternative Marginal Model

Proc Mixed syntax for the marginal model:

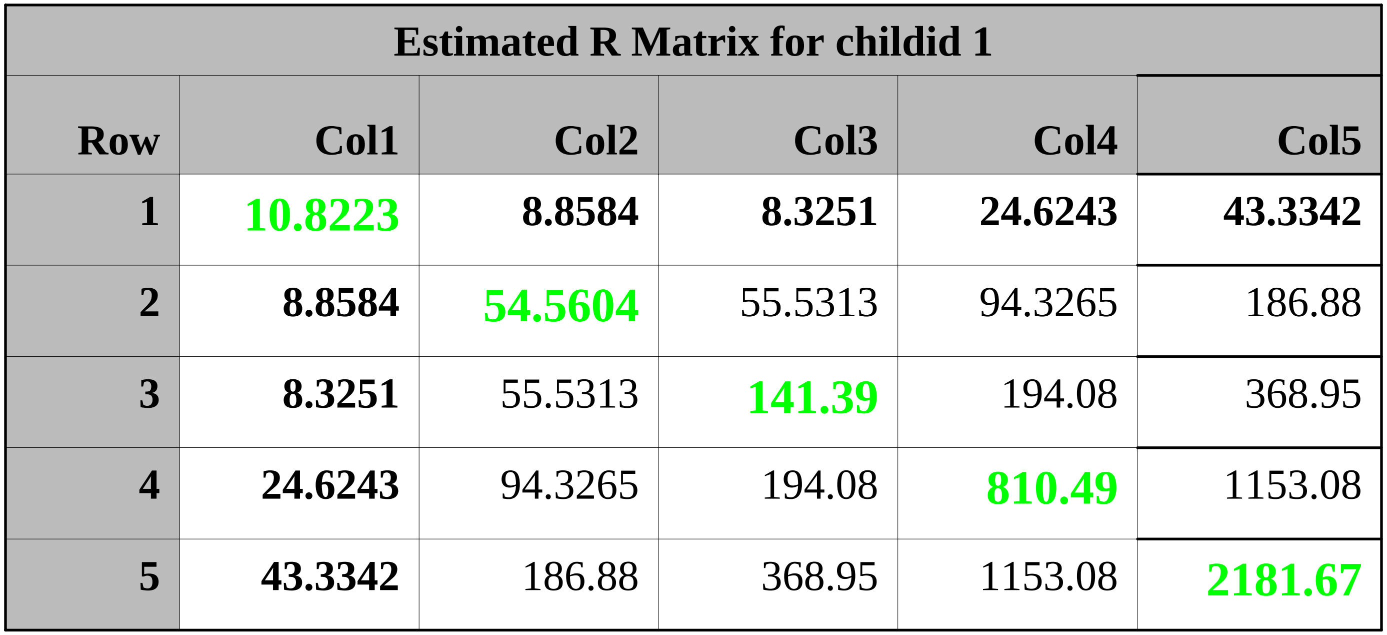
```
proc mixed data=autism2 noclprint;
   class childid sicdegp age;
   model vsae = sicdegp age_2 age_2sq age_2*sicdegp
        age_2sq*sicdegp
        / solution ddfm=bw;
   repeated age / subject=childid type=un r rcorr;
run;
```

Note: there is no random statement in this model, and hence, the ***G*** matrix = ***0***.

The repeated statement specifies that the ***R*** matrix should be unstructured.

# Marginal Model: ***R*** Matrix

Note: This ***R*** matrix shows positive covariance, and hence, positive correlation, between residuals at baseline and later ages.



**Estimated R Matrix for childid 1**

| Row | Col1 | Col2 | Col3 | Col4 | Col5 |
|---:|---:|---:|---:|---:|---:|
| 1 | 10.8223 | 8.8584 | 8.3251 | 24.6243 | 43.3342 |
| 2 | 8.8584 | 54.5604 | 55.5313 | 94.3265 | 186.88 |
| 3 | 8.3251 | 55.5313 | 141.39 | 194.08 | 368.95 |
| 4 | 24.6243 | 94.3265 | 194.08 | 810.49 | 1153.08 |
| 5 | 43.3342 | 186.88 | 368.95 | 1153.08 | 2181.67 |

Again, the variance increases at each time point, as was apparent in the initial graphs, and in the LMM.

# Model Fit Comparison

| | Full LMM | Revised LMM minus random intercept | Marginal Model |
|---|---|---|---|
| AIC | 4616.7 | 4623.3 | 4459.5 |
| BIC | 4635.1 | 4635.5 | 4505.4 |

From this comparison, the marginal model is preferable to the two LMMs for this data set. It has the smallest AIC and BIC.

# Summing Up

- The ability to fit a wide array of different covariance structures gives Proc Mixed a lot of flexibility
- By examining the ***G*** matrix, ***R*** matrix and ***V*** matrix we can see how different structures affect the model covariance parameters.
- A LMM may not always be the best choice.
- At times, a marginal model may give a better fit.
- Research goals can help in the choice of the “right” modeling approach.

# References

- Brady West, Kathleen Welch and Andrzej Galecki, “Linear Mixed Models: A Practical Guide Using Statistical Software”, Chapman & Hall/CRC, 1986, 353 pp.
- Oti, R., Anderson, D., and Lord, C. Social trajectories among individuals with autism spectrum disorders, Journal of Developmental Psychopathology.