Bayesian Analysis: A Deep Dive into BGLIMM

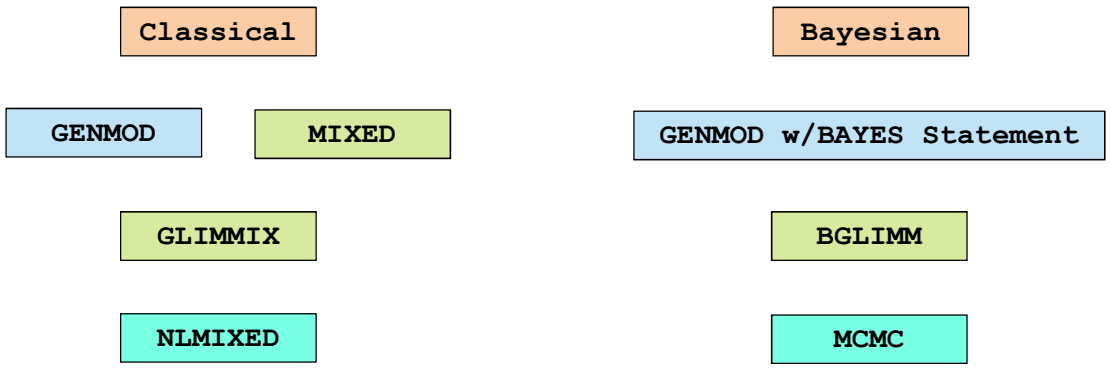
Danny Modlin

Sr. Analytical Training Consultant – Cary, NC

Objectives

- Discuss current incorporation methods for random effects.
- Review syntax of PROC BGLIMM.
- Explore code examples of PROC BGLIMM.

Mixed Modeling Procedures in SAS



General Linear Mixed Model

$$y = \mathrm{X}\beta + \mathrm{Z}\gamma + \varepsilon$$

where y is the vector of observed responses.

X is the design matrix of predictor variables.

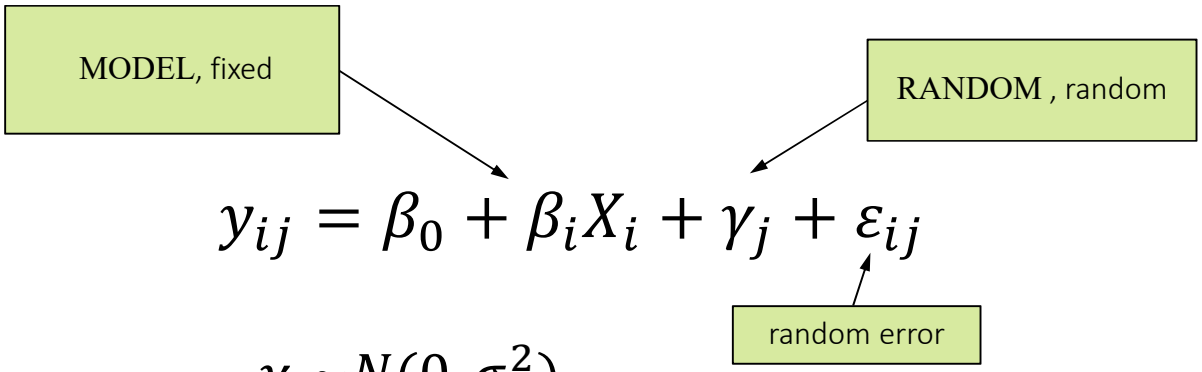
β is the vector of regression parameters.

Z is the design matrix of random variables.

γ is the vector of random-effect parameters.

ε is the vector of random errors.

General Linear Mixed Model



$$\gamma_j \sim N(0, \sigma_\gamma^2)$$

$$\varepsilon_{ij} \sim N(0, \sigma^2)$$

Generalized Linear Mixed Models (GzLMMs)

- GzLMMs enable modeling random effects and correlated errors for nonnormal data.
  - A linear predictor can contain random effects.

    $\eta = X\beta + Z\gamma$
  - The random effects are normally distributed.
  - The conditional mean, $\mu|\gamma$, relates to the linear predictor through a link function.

    $g(\mu|\gamma) = \eta$
  - The conditional distribution (given γ) of the data belongs to the exponential family of distributions.

GzLMM Formulation and PROC GLIMMIX

$$g(\mu \mid \gamma) = \mathbf{X}\beta + \mathbf{Z}\gamma$$

LINK= option | MODEL statement | RANDOM statement

$Y|\gamma$ ~ exponential family ← DIST= option

var(γ) = G ← Options in the RANDOM statement

Var($y|\ \gamma$)=$A_\mu R A_\mu$

RANDOM _RESIDUAL_ statement

***REPEATED in BGLIMM**

PROC MIXED Program

```
proc mixed data=sasuser.toy;
   class adhesive toy;
   model pressure=adhesive / solution ddfm=kr;
   random toy;
run;
```

PROC MCMC Program

```
proc mcmc data=toy seed=27513 diag=all dic outpost=mixed
     propcov=quanew thin=25 nbi=5000 ntu=5000 nmc=500000
     plots(smooth)=all mchistory=brief stats=all;
   array beta[3];
   parms beta: 0;
   parms s2t 1;
   parms s2g 1;
   prior beta: ~ normal(0, var = 1e5);
   prior s2: ~ igamma(2.001, scale = 1.001);
   random gamma ~ normal(0,var=s2g) subject=toy
          monitor=(gamma) namesuffix=position;
   mu = beta[adhesivebeta] + gamma;
   model pressure ~ normal(mu, var = s2t);
   title "Bayesian Analysis of the Toy Data Set";
run;
```


PROC BGLIMM Program

```
proc bglimm data=sasuser.toy seed=8675309;
   class adhesive toy;
   model pressure=adhesive / dist=normal;
   random int / sub=toy;
run;
```

Features of BGLIMM

- Suite of covariance structures (for both G- and R-side)
- Covariance heterogeneity modeling
- Built-in priors
- Model Comparison via DIC statistic
- Multi-threading of optimal sampling

PROC BGLIMM Syntax

PROC BGLIMM Statement

- DATA= names the input data set
- SEED= random seed for simulation
- OUTPOST= output a data set to contain posterior samples
- NBI= number of burn-in iterations
- NMC= number of Markov chain iterations
- NTHREADS= number of CPUs to run simulations simultaneously
- STATS= posterior statistics
- DIAG= convergence diagnostics
- PLOTS= plotting

PROC BGLIMM Syntax

MODEL response = fixed effects / dist= link= …;

- 9 response distributions:
  - Binomial
  - Exponential
  - Gamma
  - Geometric
  - Inverse Gaussian
  - Negative binomial
  - Normal
  - Poisson
  - Binary
- 8 link functions:
  - Log
  - Logit
  - Probit
  - Inverse
  - Identity
  - Loglog
  - Complementary loglog
  - PowerMinus2

PROC BGLIMM Syntax

RANDOM random-effects / sub= group= type= …;

- SUB= option to identify the subjects for the random effects
- GROUP= option to identify groups by which to vary the covariance parameters; each new level of the grouping effect produces a new set of covariance parameters
- TYPE= option to define the covariance structure of G
  - 13 choices: AR, ARMA, CS, TOEP, UN, VC, …
- *Multiple RANDOM statements can be used.

PROC BGLIMM Syntax

REPEATED repeated-effect / sub= group= type= …;

- A repeated-effect is required to define the proper location of the repeated responses.
- SUB= option to group repeated measures together for the same subject
- GROUP= option to identify groups by which to vary the covariance parameters
- TYPE= option to define the covariance structure
  - 13 choices: AR, ARMA, CS, TOEP, UN, VC, …

Sampling

- PROC BGLIMM updates parameters conditionally and sequentially through Gibbs sampling.
  - The fixed-effect parameters are drawn together first at each iteration.
  - The random-effect parameters are updated by subjects.
  - The G-side covariance parameters are then sampled.
  - Lastly, the R-side covariance parameters are updated.
  - If present, missing response values are treated as parameters and are thus sampled as well.

Prior Distributions

- Fixed-effect parameters (Betas)
  - Flat/constant; normal
- Scale parameter
  - Inverse gamma; gamma; improper
- G-side Covariance parameters
  - Inverse wishart; inverse gamma; uniform; halfcauchy; halfnormal; siwishart
- R-side Covariance parameters
  - Inverse wishart; inverse gamma

Examples using BGLIMM

This demonstration illustrates the concepts discussed previously.

Questions?