

# Introduction to Proc MI and Proc MIAnalyze for Multiple Imputation of Missing Data in SAS

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Michigan SAS Users Group

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# Missing Data is a Problem in Many Studies

- Attrition in longitudinal studies
  - Drug doesn't work/ side effects
  - Drug works too well
- Item non-response in surveys
  - Don't answer questions on income, but answer other questions
- Partial non-response
  - Participate in interview, but not physical exam

# Missing Data Mechanisms

- MCAR: missing completely at random, missingness (M) has no relationship to data (Y)  
 $p(M|Y) = p(M)$ , for all Y
- MAR: Missing At Random, missingness only related to observed data values  
 $p(M|Y) = p(M|Y_{\text{obs}})$ , for all Y
- NMAR: Not Missing at random, missingness depends on the ***unobserved*** values of Y
  - AKA, informative missing, non-ignorable missing
- Rubin, 1976, Little and Rubin, 2002

# What to do if NMAR

- If data are not missing at random, the missing data *mechanism* needs to be modeled
- There are methods for this, but not implemented in standard software packages

# Missing Data Patterns

- General pattern:

$Y_1$	$Y_2$	$Y_3$

- Monotone pattern:

- When one value is missing, all subsequent values are missing

$Y_1$	$Y_2$	$Y_3$

# Some methods for handling missing data

- CC: Complete Cases analysis, weighted complete case analysis
- Single Imputation methods
  - Mean imputation
  - Conditional mean imputation
- Multiple Imputation
- Other methods, e.g., bootstrap imputations
- All methods have limitations

# Complete Case Analysis

- Throw away cases with any missing data
- Default method in most SAS procedures (e.g., Proc Reg, Proc GLM)
- Possible problems
  - Can result in serious bias, if missing cases are not MCAR
  - Variances of estimates will be greater than if all cases were used
- May be fine, esp. if amount of missing data is small
- Include in analysis variables that predict missingness (Allison)

# Single Imputation Methods

- Use incomplete data to get a plausible predicted value for the missing data
- Create a “complete” data set for analysis
- Advantage: Once through the data
- But can have problems
  - May be seriously biased
  - May Understate the variability of estimates



# Mean Imputation

- Replace missing data for continuous variables by Mean of non-missing values
- Replace missing data for categorical variables by the mode
- Because marginal distributions are used, associations are distorted
- Standard deviations of completed data are too small (variability is artificially reduced)
- Sample size is overestimated
- Can be less efficient than CC analysis
- Can result in serious bias

# Conditional Mean Imputation

- Conditional on observed values
- Use a regression model to predict the missing values
- Better than unconditional mean, more plausible values
- Standard errors of estimates are still too small
- CC analysis may be better

# Multiple Imputation

- Create  $m$  sets of imputations (complete data)
- Analyze the  $m$  sets of imputations with usual statistical methods
- Combine estimates to get imputation inference
- Very useful for large public datasets, so users can analyze complete data

# Number of Imputations

- Relative efficiency of a small number (m) of imputations compared to the theoretical infinite number of imputations is high
- 5 imputations the default for Proc MI

Relative Efficiencies\*

$\lambda$					
m	10%	20%	30%	50%	70%
3	0.9677	0.9375	0.9091	0.8571	0.8108
5	0.9804	0.9615	0.9434	0.9091	0.8772
10	0.9901	0.9804	0.9709	0.9524	0.9346
20	0.9950	0.9901	0.9852	0.9756	0.9662

\*Table 44.5 from SAS documentation for Proc MI, SAS 9.2

# Proper Multiple Imputation Methods

- Each dataset is a random draw from the joint posterior predictive distribution of missing values and parameters of the distribution
- The uncertainty in estimating  $\theta$  is taken into account
  - Propagate the error in estimating  $\theta$  in the imputations (this is good)
- Estimates have high efficiency
- The accuracy increases with  $m$ , and the percent of complete data

# Imputation Model vs. Analysis Model

- Imputation model may use more variables than analysis model
- This is OK
  - Imputation may use many more variables, even though not of substantive interest
  - Different analysts may want to use different predictors in their models
- Improved efficiency and less bias by using more variables for imputation model
- But try to use variables in imputation that are at least helpful for imputation of missing values

# Combining Estimates from Multiple Imputation

$$\bar{\Theta} = \frac{1}{M} \sum_{m=1}^M \hat{\Theta}_m$$

Combined Parameter Estimate

$$T = \bar{W} + (1 + \frac{1}{M})B$$

Total variance

$$\bar{W} = \frac{1}{M} \sum_{m=1}^M W_m$$

Within-imputation variance

$$B = \frac{1}{M-1} \sum_{m=1}^M (\hat{\Theta}_m - \bar{\Theta})^2$$

Between-imputation variance

# Combining Estimates (Cont)

Fraction of Missing Information

$$r = \frac{(1 + 1 / m)B}{\bar{W} + (1 + 1 / m)B}$$

Degrees of Freedom

$$\nu = (m - 1) / r^2$$

Degrees of Freedom are increased by having more imputations, or less missing data



# SAS Proc MI Methods for Multiple Imputation

- Methods available in Proc MI depend on missing data pattern
- For arbitrary missing data pattern use Markov-chain Monte Carlo (MCMC) method
  - Based on Bayes methods
  - Proper Imputation
  - Assumes a multivariate normal distribution
  - This may be problematic, esp. for missing categorical variables, but not too bad
- Can be used to get to a monotone missing data pattern, then another method, such as logistic regression for categorical variables, may be used

# Proc MI Methods Available for Different Missing Data Patterns

Pattern of Missingness	Type of Imputed Variable	Recommended Methods
Monotone	Continuous	Regression Predicted Mean Matching Propensity Score
Monotone	Classification (Ordinal)	Logistic Regression
Monotone	Classification (Nominal)	Discriminant Function Method
Arbitrary	Continuous	MCMC Full-Data Imputation MCMC Monotone-Data Imputation

Table 44.3 from Proc MI documentation, SAS 9.1.3

# Example Using Proc MI and MIAnalyze

- LSOA study (Longitudinal Study of Aging, NCHS)

<http://www.cdc.gov/nchs/about/otheract/aging/lsoa1.htm>

- Baseline data collected in 1984, follow-up in 1986.
- 5151 participants, aged 70 to 99 in 1984
- 64% female
- Want to model changes in ADL (restrictions in Activities of Daily Living) from 1984 to 1986
- Missing data on over 21% for ADL in 1986

# Variables in LSOA Dataset

## Variables in Creation Order

#	Variable	Type	Len	Label
1	SEX	Num	3	1=MALE 2=FEMALE
2	AGE84	Num	3	Age during 1984
3	RACER	Num	3	1=WHITE 2=BLACK 3=OTHER
4	EDUC	Num	3	Education of individual-completed years
5	POVERTY	Num	3	NHIS poverty index
6	OWNBUYR	Num	3	Own/buying recode
7	MORTGAGE	Num	3	Fully paid for or mortgage being paid
8	AMTOWED	Num	4	Amount principal still owed
9	PRESVAL	Num	4	Present value of place
10	NUMADL	Num	3	Number of ADLs, 1984
11	NUMADL2R	Num	3	Number of ADLs, 1986
12	FNLWGT2	Num	3	Final 1986 LSOA weight
13	STRATUM	Num	8	
14	PSU	Num	8	

# LSOA Analysis Plan

- Check pattern of missing data
- Create five multiply imputed datasets using Proc MI
- Carry out a regression analysis of the five datasets using Proc Reg
- Combine the estimates from the five datasets to get the MI estimates, using Proc MIAnalyze
- Carry out a CC analysis for comparison

# LSOA Descriptives for Original Dataset

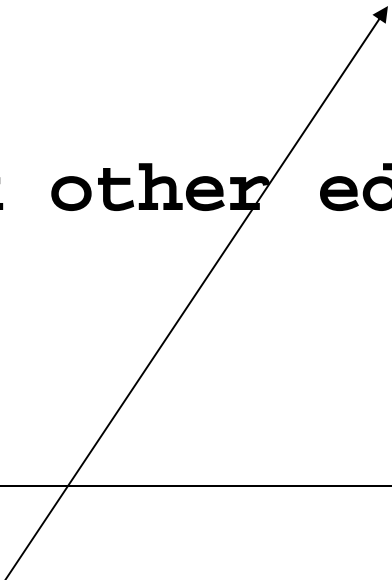
The MEANS Procedure

Variable	N	N Miss	Mean	Std Dev
diffadl	4048	1103	0.2197777	0.5834531
POVERTY	4229	922	1.1941357	0.3955806
own	5015	136	0.6638086	0.4724524
EDUC	5053	98	9.7573719	3.7452672
AGE84	5151	0	78.2071442	6.0010675
SEX	5151	0	1.6396816	0.4801394
black	5151	0	0.1087168	0.3113137
other	5151	0	0.0108717	0.1037091

Much missing data on the outcome variable

# Proc MI SAS Code

```
proc mi data=lsoa nimpute=5 out=OUTMI  
seed=3355;  
  
var poverty own sex black other educ  
age84 logadl logadl2r ;  
  
run;
```



OUTMI contains the five multiply-imputed datasets  
\_IMPUTATION\_ variable indexes imputations

# Proc MI Model Information

## The MI Procedure

### Model Information

Data Set	WORK.LSOA
Method	MCMC
Multiple Imputation Chain	Single Chain
Initial Estimates for MCMC	EM Posterior Mode
Start	Starting Value
Prior	Jeffreys
Number of Imputations	5
Number of Burn-in Iterations	200
Number of Iterations	100
Seed for random number generator	3355



# Proc MI Missing Data Patterns

Missing Data Patterns

Group	POVERTY	own	SEX	black	other	EDUC	AGE84	logadl	logadl2r	Freq
1	X	X	X	X	X	X	X	X	X	3268
2	X	X	X	X	X	X	X	X	.	791
3	X	X	X	X	X	X	X	.	X	7
4	X	X	X	X	X	X	X	.	.	7
5	X	X	X	X	X	.	X	X	X	28
6	X	X	X	X	X	.	X	X	.	27
7	X	X	X	X	X	.	X	.	X	1
8	X	.	X	X	X	X	X	X	X	77
9	X	.	X	X	X	X	X	X	.	21
10	X	.	X	X	X	.	X	X	X	2
11	.	X	X	X	X	X	X	X	X	624

# Proc MI Missing Data Patterns: Group Means

Missing Data Patterns					
Group	Percent	-----Group Means-----			
		POVERTY	own	SEX	black
1	63.46	1.186047	0.686965	1.646879	0.109241
2	15.36	1.231353	0.596713	1.573957	0.113780
3	0.14	1.285714	0.857143	1.571429	0
4	0.14	1.142857	0.571429	1.428571	0.142857
5	0.54	1.178571	0.571429	1.678571	0.178571
6	0.52	1.259259	0.370370	1.666667	0.222222
7	0.02	1.000000	1.000000	2.000000	0
8	1.50	1.155844	.	1.532468	0.077922
9	0.41	1.142857	.	1.476190	0.095238
10	0.04	1.000000	.	1.500000	0.500000
11	12.12	.	0.692308	1.684295	0.105769

# Proc MI Variance Information

## Multiple Imputation Variance Information

Variable	-----Variance-----			DF
	Between	Within	Total	
POVERTY	0.000001824	0.000030417	0.000032606	749.29
own	0.000000864	0.000043349	0.000044386	2981.6
EDUC	0.000022363	0.002728	0.002755	4547.1
logad1	0.000000234	0.000070284	0.000070565	5024.8
logad12r	0.000024269	0.000098270	0.000127	75.09

## Multiple Imputation Variance Information

Variable	Relative Increase in Variance	Fraction Missing Information	Relative Efficiency
POVERTY	0.071951	0.069216	0.986346
own	0.023922	0.023630	0.995296
EDUC	0.009836	0.009787	0.998046
logad1	0.003991	0.003983	0.999204
logad12r	0.296360	0.248006	0.952743

# Proc MI Parameter Estimates

## Multiple Imputation Parameter Estimates

Variable	Mean	Std Error	95% Confidence Limits		DF
POVERTY	1.194470	0.005710	1.183260	1.205680	749.29
own	0.664994	0.006662	0.651931	0.678057	2981.6
EDUC	9.743557	0.052489	9.640652	9.846461	4547.1
logadl	0.349539	0.008400	0.333070	0.366007	5024.8
logadl2r	0.559964	0.011287	0.537480	0.582448	75.09

## Multiple Imputation Parameter Estimates

Variable	Minimum	Maximum	Mu0	t for H0:	
				Mean=Mu0	Pr >  t
POVERTY	1.193273	1.196416	0	209.18	<.0001
own	0.663869	0.665959	0	99.81	<.0001
EDUC	9.738145	9.749231	0	185.63	<.0001
logadl	0.348811	0.350080	0	41.61	<.0001
logadl2r	0.554439	0.567925	0	49.61	<.0001

# Modify Data and Check Means

```
/*Create Difference on Log Scale AFTER Imputation*/  
data OUTMI;  
  set OUTMI;  
  diffadl = logadl2r - logadl;  
run;  
proc means data=outmi;  
by _imputation_;  
run;
```

# Descriptives for Imputation 1

----- \_Imputation\_=1 -----

The MEANS Procedure

Variable	N	N Miss	Minimum	Maximum	Mean
SEX	5150	0	1.0000000	2.0000000	1.6398058
AGE84	5150	0	70.0000000	99.0000000	78.2077670
RACER	5150	0	1.0000000	3.0000000	1.1304854
EDUC	5150	0	-0.6051802	18.0000000	9.7473659
POVERTY	5150	0	-0.3943656	2.7625847	1.1964162
own	5150	0	-0.7833835	1.8704920	0.6651582
logadl2r	5150	0	-1.1814288	2.8560990	0.5544391
logadl	5150	0	-0.2376750	2.0794415	0.3498080
black	5150	0	0	1.0000000	0.1087379
other	5150	0	0	1.0000000	0.0108738
ownmiss	5150	0	0	1.0000000	0.0264078
povmiss	5150	0	0	1.0000000	0.1788350
diffadl	5150	0	-2.0794415	2.3073451	0.2046311

-----

# Descriptives for Imputation 2

----- _Imputation_=2 -----					
Variable	N	N Miss	Minimum	Maximum	Mean
SEX	5150	0	1.0000000	2.0000000	1.6398058
AGE84	5150	0	70.0000000	99.0000000	78.2077670
RACER	5150	0	1.0000000	3.0000000	1.1304854
EDUC	5150	0	0	18.0000000	9.7492314
POVERTY	5150	0	-0.0334959	2.4663084	1.1953344
own	5150	0	-0.8350119	2.0972619	0.6657782
logadl2r	5150	0	-1.6669717	3.1120230	0.5679253
logadl	5150	0	-0.6301672	2.0794415	0.3500805
black	5150	0	0	1.0000000	0.1087379
other	5150	0	0	1.0000000	0.0108738
ownmiss	5150	0	0	1.0000000	0.0264078
povmiss	5150	0	0	1.0000000	0.1788350
diffadl	5150	0	-2.0794415	2.1250189	0.2178448
-----					

# SAS Code for Proc Reg

```
proc reg data=OUTMI outest=OUTREG covout ;  
  by _Imputation_;  
  model diffadl=poverty own educ logadl age84 sex black other;  
run;
```



# Output from Regression for Imputation 1

The REG Procedure  
Model: MODEL1  
Dependent Variable: diffad1

Number of Observations Read 5150  
Number of Observations Used 5150

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr >  t
Intercept	1	-1.06429	0.11564	-9.20	<.0001
POVERTY	1	0.00564	0.02052	0.27	0.7836
own	1	-0.04346	0.01666	-2.61	0.0091
EDUC	1	-0.00714	0.00220	-3.25	0.0012
logad1	1	-0.33447	0.01335	-25.06	<.0001
AGE84	1	0.01892	0.00135	14.05	<.0001
SEX	1	-0.00591	0.01627	-0.36	0.7163
black	1	0.08336	0.02585	3.22	0.0013
other	1	-0.13478	0.07383	-1.83	0.0680

# Output from Regression for Imputation 2

The REG Procedure  
Model: MODEL1  
Dependent Variable: diffad1

Number of Observations Read 5150  
Number of Observations Used 5150

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr >  t
Intercept	1	-1.11195	0.11696	-9.51	<.0001
POVERTY	1	0.04296	0.02096	2.05	0.0405
own	1	-0.04476	0.01688	-2.65	0.0080
EDUC	1	-0.00755	0.00223	-3.39	0.0007
logad1	1	-0.33505	0.01350	-24.83	<.0001
AGE84	1	0.01895	0.00136	13.89	<.0001
SEX	1	0.00683	0.01648	0.41	0.6784
black	1	0.07137	0.02622	2.72	0.0065
other	1	-0.16516	0.07474	-2.21	0.0272

# Proc MIAnalyze Code to Combine Estimates Across Imputations

```
proc mianalyze data=OUTREG;  
  modeleffects Intercept poverty own educ logadl  
  age84 sex black other;  
run;
```

# Proc MIAnalyze Output

## The MIANALYZE Procedure

### Model Information

Data Set	WORK.OUTREG
Number of Imputations	5

### Multiple Imputation Variance Information

Parameter	-----Variance-----			DF
	Between	Within	Total	
Intercept	0.001782	0.013592	0.015730	216.5
poverty	0.000324	0.000432	0.000821	17.816
own	0.000310	0.000282	0.000655	12.358
educ	0.000000724	0.000004891	0.000005760	175.62
logadl	0.000112	0.000181	0.000315	21.953
age84	0.000000485	0.000001842	0.000002424	69.323
sex	0.000073830	0.000269	0.000357	65.112
black	0.000175	0.000680	0.000890	71.717
other	0.002603	0.005532	0.008655	30.722

# Proc MIAnalyze Output (Cont)

## Multiple Imputation Parameter Estimates

Parameter	Estimate	Std Error	95% Confidence Limits		DF
Intercept	-1.081213	0.125421	-1.32842	-0.83401	216.5
poverty	0.035685	0.028650	-0.02455	0.09592	17.816
own	-0.031520	0.025584	-0.08708	0.02404	12.358

## Multiple Imputation Parameter Estimates

Parameter	Minimum	Maximum
Intercept	-1.132226	-1.024379
poverty	0.005635	0.053830
own	-0.044841	-0.010716

## Multiple Imputation Parameter Estimates

t for H0:

Parameter	Theta0	Parameter=Theta0	Pr >  t
Intercept	0	-8.62	<.0001
poverty	0	1.25	0.2291
own	0	-1.23	0.2409

# Proc MIAnalyze Output (Cont)

Multiple Imputation Parameter Estimates					
Parameter	Estimate	Std Error	95% Confidence Limits		DF
educ	-0.008150	0.002400	-0.01289	-0.00341	175.62
logadl	-0.341935	0.017754	-0.37876	-0.30511	21.953
age84	0.018647	0.001557	0.01554	0.02175	69.323
sex	0.002601	0.018906	-0.03516	0.04036	65.112
black	0.070117	0.029827	0.01065	0.12958	71.717
other	-0.123315	0.093033	-0.31313	0.06650	30.722

Parameter	Minimum	Maximum
educ	-0.009317	-0.007142
logadl	-0.360206	-0.334471
age84	0.017851	0.019504
sex	-0.005912	0.014089
black	0.049046	0.083361
other	-0.165156	-0.034706

t for H0:			
Parameter	Theta0	Parameter=Theta0	Pr >  t
educ	0	-3.40	0.0008
logadl	0	-19.26	<.0001
age84	0	11.98	<.0001
sex	0	0.14	0.8910
black	0	2.35	0.0215
other	0	-1.33	0.1948

# Impute Values for Poverty (Binary) Using Proc Logistic on Imputed Data

```
data SIX;
  set OUTMI;
  if povmiss=1 then poverty = .;
run;
proc mi data=SIX nimpute=1 out=OUTMI2 seed=3355;
  by _Imputation_;
  class POVERTY;
  monotone logistic (POVERTY = own educ logadl logadl2r age84
sex black other / details);
  var own educ logadl logadl2r age84 sex black other poverty;
run;
proc reg data=OUTMI2 outest=OUTREG2 covout ;
  by _imputation_;
  model diffadl=poverty own educ logadl age84 sex black other;
run;
```

# Output from Complete Case Analysis

The REG Procedure  
Model: MODEL1  
Dependent Variable: diffad1

Number of Observations Read	5151
Number of Observations Used	3268
Number of Observations with Missing Values	1883

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	8	121.35294	15.16912	50.43	<.0001
Error	3259	980.21561	0.30077		
Corrected Total	3267	1101.56855			

Root MSE	0.54843	R-Square	0.1102
Dependent Mean	0.20687	Adj R-Sq	0.1080
Coeff Var	265.10481		

Proc MI and Proc MIAnalyze  
Michigan SUG: Kathy Welch



# Compare CC vs. MI Results

	CC Analysis				Proc MI			
Variable	Estimate	SE	df	p-value	Estimate	SE	df	p-value
Int	-0.84897	0.14929	3259	<.0001	-1.081213	0.125421	216.5	<.0001
Poverty	0.03078	0.02626	3259	0.2412	0.035685	0.028650	17.816	0.2291
Own	-0.01959	0.02131	3259	0.3580	-0.031520	0.025584	12.358	0.2409
Educ	-0.00922	0.00278	3259	0.0009	-0.008150	0.002400	175.62	0.0008
Logadl	-0.34402	0.01836	3259	<.0001	-0.341935	0.017754	21.953	<.0001
Age84	0.01631	0.00175	3259	<.0001	0.018647	0.001557	69.323	<.0001
Sex	-0.02703	0.02056	3259	0.1886	0.002601	0.018906	65.112	0.8910
Black	0.03811	0.03223	3259	0.2372	0.070117	0.029827	71.717	0.0215
Other	-0.08840	0.09632	3259	0.3588	-0.123315	0.093033	30.722	0.1948

# Summing Up

- There are many methods for analyzing missing data with missing values
- No method is perfect
- CC analysis is better than some mean imputation methods
- Multiple imputation is better than single imputation
- Uncertainty in parameter estimates is correctly maintained in MI methods

# References

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# References (Cont)

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